Stop!

Do not continue until directed to do so!
Round #1
(10 minutes)
Problem A (4 points)
Kimo paddles upstream for 8 hours. If his rate of paddling is twice the rate of the current, at what time will he have to start heading downstream in order to reach his starting point at 6pm?

Problem B (5 points)
A math professor, basketball coach, and their student went to Burger Queen for lunch. The math professor paid $14.25 for 8 hamburgers, 5 orders of fries, and 2 cokes. The basketball coach paid $8.51 for 5 hamburgers, 3 orders of fries, and 1 coke. What did the student pay for 1 hamburger, 1 order of fries, and 1 coke?

Problem C (6 points)
Find all real solutions for $x$:

$\left( x^2 + 2x - 24 \right) x^3 - 9x^2 + 20x = 1$
Round #1
(10 minutes)

Answer: Problem A (4 points)

Answer: Problem B (5 points)

Answer: Problem C (6 points)
Answer: Problem A (4 points)
3:20 pm

Answer: Problem B (5 points)
$2.97

Answer: Problem C (6 points)
$0, 5, -1 \pm \sqrt{26}$
Stop!
Do not continue until directed to do so!
Round #2

(5 points/5 minutes)
A circle has both inscribed and circumscribed squares. The difference of their areas is 40 square units. What is the circumference of the circle?
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Answer:
A circle has both inscribed and circumscribed squares. The difference of their areas is 40 square units. What is the circumference of the circle?

Answer:

$$4\sqrt{5}\pi$$
Solution:

As seen in the diagram, the area of the circumscribed square is $(2r)^2 = 4r^2$. The inscribed square consists of four right triangles with legs of length $r$. Thus, the area of the inscribed square is $4\left(\frac{1}{2}r^2\right) = 2r^2$. The difference between the areas is 40. We can solve for $r$ like so:

\[
4r^2 - 2r^2 = 40 \implies 2r^2 = 40
\]

\[
\implies r^2 = 20 \implies r = \sqrt{20} = 2\sqrt{5}
\]

It follows that the circumference is

\[
C = 2\pi r = 2\pi(2\sqrt{5}) = 4\sqrt{5}\pi
\]
Stop!
Do not continue until directed to do so!
Round #3

(6 points/6 minutes)
Chris and Pat work at Hawaiian Electric. One day, Chris said to Pat, “19/40 of my coworkers are female”. Pat replied “12/25 of my coworkers are female”. If Chris and Pat are of different genders and they are both correct, then how many workers are there at Hawaiian Electric?
Chris and Pat work at Hawaiian Electric. One day, Chris said to Pat, “19/40 of my coworkers are female”. Pat replied “12/25 of my coworkers are female”. If Chris and Pat are of different genders and they are both correct, then how many workers are there at Hawaiian Electric?

Answer: 201 workers
Chris and Pat work at Hawaiian Electric. One day, Chris said to Pat, “19/40 of my coworkers are female”. Pat replied “12/25 of my coworkers are female”. If Chris and Pat are of different genders and they are both correct, then how many workers are there at Hawaiian Electric?

Answer: 201 workers
Solution:

Pat and Chris have the same number of co-workers. However, they differ in female co-workers by 1. Since
\[
\frac{19}{40} = 0.475 < \frac{12}{25} = 0.48,
\]
then Chris must be female. Suppose there are \( w \) co-workers. Then it follows that

\[
\frac{f}{w} = \frac{19}{40} \Rightarrow f = \frac{19}{40}w
\]
and

\[
\frac{f + 1}{w} = \frac{12}{25} \Rightarrow 25(f + 1) = 12w
\]

\[
25\left(\frac{19}{40}w + 1\right) = 12w
\]

\[
\frac{95}{8}w + 25 = 12w
\]

\[
-\frac{1}{8}w = -25
\]

\[
w = 200
\]

Because either Chris or Pat would have 200 co-workers, there is a total of \(w + 1 = 201\) workers.
Stop!

Do not continue until directed to do so!
Round #4
(7 points/7 minutes)
An army of ants is marching across the kitchen floor. If they form columns with 10 ants in each column, then there are 6 ants left over. If they form columns of 7, 11, or 13 ants in each column, then there would be 2 ants left over. What is the smallest number of ants that could be in this army?

Answer:

4006 ants
An army of ants is marching across the kitchen floor. If they form columns with 10 ants in each column, then there are 6 ants left over. If they form columns of 7, 11, or 13 ants in each column, then there would be 2 ants left over. What is the smallest number of ants that could be in this army?

**Answer:**

4006 ants
An army of ants is marching across the kitchen floor. If they form columns with 10 ants in each column, then there are 6 ants left over. If they form columns of 7, 11, or 13 ants in each column, then there would be 2 ants left over. What is the smallest number of ants that could be in this army?

Answer:

4006 ants
Solution:

Let $t$ be the total number of ants. Because groups of 7, 11, or 13 produce two ants left over, $t - 2$ is a multiple of 7, 11, and 13. Hence $t - 2$ is a multiple of the least common multiple of 7, 11, and 13; because those numbers are relatively prime, $t - 2$ is therefore a multiple of $7 \times 11 \times 13 = 1001$. We are also given that groups of 10 ants produce 6 ants left over, so the last digit of $t$ is 6. Given that $t$ is 2 more than a multiple of 1001, the first $t$ to satisfy all these conditions is 4006.
Stop!

Do not continue until directed to do so!
Round #5
(8 minutes)
Problem A (Algebra-Plane Geometry/6 points)

A can of macadamia nuts has a 3 inch diameter and 4 inch height. What is the maximum number of cans that will fit in a box that measures 24 inches long, 24 inches wide, and 4 inches high on the inside?

Problem B (Trigonometry-Analytical Geometry/6 points)

Triangle $ABC$ is an isosceles right triangle with legs $AC = BC = 1$. Triangle $ADC$ is an equilateral triangle with sides of length 1. If $E$ is the point of intersection of $AB$ and $CD$, what is the area of triangle $ADE$?

Problem C (Probability-Statistics/6 points)

The mean score for those who passed the last test in Dr. Furuto’s class was 65, while the mean score for those who failed the test was 35. If the mean for the entire class was 53, what percentage of the class passed the test?
### Answer: Problem A (6 points)

### Answer: Problem B (6 points)

\[
\sqrt{3} - \frac{3}{4}
\]

### Answer: Problem C (6 points)

60\%
Answer: Problem A (6 points)

68

Answer: Problem B (6 points)

\[ \frac{\sqrt{3}}{2} - \frac{3}{4} \quad \text{or} \quad \frac{2\sqrt{3} - 3}{4} \]

Answer: Problem C (6 points)

60%
Stop!

Do not continue until directed to do so!
Round #6
(9 points/9 minutes)
Find a four digit integer, $x$, such that $4x$ is another four-digit number whose digits are in the reverse order of the digits of $x$. 

Answer: 2178
Find a four digit integer, $x$, such that $4x$ is another four-digit number whose digits are in the reverse order of the digits of $x$.

Answer: 2178
Find a four digit integer, $x$, such that $4x$ is another four-digit number whose digits are in the reverse order of the digits of $x$.

**Answer:**

2178
Solution:
Suppose the four digit number is $ABCD$. Then

$$4(1000A + 100B + 10C + D) = 1000D + 100C + 10B + A$$

$$1333A + 130B = 20C + 332D$$

If $A$ is 3 or larger, then both $C$ and $D$ could not be single digits. Also, the coefficients of $B$, $C$, and $D$ are even, so $A$ cannot be odd. Therefore $A$ is 2. Substituting and simplifying further yields

$$1333 + 65B = 10C + 166D$$

Because the coefficients of $C$ and $D$ are both even, then the sum on the left must be even; thus $B$ must be odd. When trying different possible values of $B$, $D$ can be determined quickly because $C$ cannot affect the ones’ digit of the sum on the right. Of the possible combinations only $B = 1$ works, where $C = 7$, and $D = 8$. 
Stop!

Do not continue until directed to do so!
Round #7
(15 minutes)
Problem A (4 points)

Suppose that $0 \leq x \leq 2\pi$. Find all solutions to the equation

$$\frac{\cos x}{\cos x + \sin x} + \frac{\sin x}{\cos x - \sin x} = \frac{\tan 2x + \cot 2x}{\sec 2x}$$

Problem B (6 points)

How many integers with 4 different digits are there between 1,000 and 9,999 such that the absolute value of the difference between the first digit and the last digit is 2?

Problem C (10 points)

Find the sum of the digits in the following product:

Round #7 (15 minutes)

Answer: Problem A (4 points)

Answer: Problem B (6 points)

Answer: Problem C (10 points)
### Round #7 (15 minutes)

**Answer: Problem A (4 points)**

\[\frac{\pi}{8},\; \frac{5\pi}{8},\; \frac{9\pi}{8},\; \frac{13\pi}{8}\]

**Answer: Problem B (6 points)**

840

**Answer: Problem C (10 points)**

90
Stop!
Do not continue until directed to do so!
Round #8
(10 points/10 minutes)
A 3-4-5 right triangle is drawn so that the endpoints of the hypotenuse are (0,0) and (5,0). A square with area 1 is drawn so that its center is the vertex of the right triangle and its sides are parallel to the x and y axes. What is the area of the region where the triangle and square overlap?
A 3-4-5 right triangle is drawn so that the endpoints of the hypotenuse are (0,0) and (5,0). A square with area 1 is drawn so that its center is the vertex of the right triangle and its sides are parallel to the x and y axes. What is the area of the region where the triangle and square overlap?

Answer:
A 3-4-5 right triangle is drawn so that the endpoints of the hypotenuse are (0,0) and (5,0). A square with area 1 is drawn so that its center is the vertex of the right triangle and its sides are parallel to the x and y axes. What is the area of the region where the triangle and square overlap?

**Answer:**

\[
\begin{array}{c}
\frac{1}{4}
\end{array}
\]
Solution:

In the figure on the right, the rectangle corresponds to the bottom half of the square of area 1, and the point $B$ corresponds to the top of the 3-4-5 triangle as described. It follows that $\triangle ABF$ and $\triangle BCD$ are 3-4-5 triangles as well.

The area of the overlap is the area of $\triangle ABF$ plus the area of the trapezoid $FBDE$. However, since $\triangle ABF$ and $\triangle BCD$ are exactly the same (they are similar triangles and one side is the same), the area of the overlap is the same as the area of the square $BCEF$, which is $\frac{1}{4}$. 
Stop!

Do not continue until directed to do so!
Tie-Breaker #1

(1 point)
Tie-Breaker #1

(1 point)

Find the tens digit in the sum

\[(1!)^3 + (2!)^3 + (3!)^3 + (4!)^3 + \cdots + (100!)^3\]
Tie-Breaker #1
(1 point)

Find the tens digit in the sum

\[(1!)^3 + (2!)^3 + (3!)^3 + (4!)^3 + \cdots + (100!)^3\]

Answer:

\[14049\]

Solution:
When 100 is a factor of a number, the number ends in two zeros and can add nothing to the tens digit of the sum. Since \((5!)^3\) has 100 as a factor (and so does every other term \((n!)^3\), where \(n > 5\)), it follows that the tens digit of the entire sum would have the same tens digit as the sum of the first four terms. Now, \((1!)^3 + (2!)^3 + (3!)^3 + (4!)^3 = 1 + 8 + 216 + 13824 = 14049\). Thus, the tens digit is 4.
Find the tens digit in the sum

\[(1!)^3 + (2!)^3 + (3!)^3 + (4!)^3 + \cdots + (100!)^3\]

Answer: 4
Find the tens digit in the sum

\[(1!)^3 + (2!)^3 + (3!)^3 + (4!)^3 + \cdots + (100!)^3\]

**Answer:**

4

**Solution:**

When 100 is a factor of a number, the number ends in two zeros and can add nothing to the tens digit of the sum. Since \((5!)^3\) has 100 as a factor (and so does every other term \((n!)^3\), where \(n > 5\)), it follows that the tens digit of the entire sum would have the same tens digit as the sum of the first four terms. Now,

\[(1!)^3 + (2!)^3 + (3!)^3 + (4!)^3 = 1 + 8 + 216 + 13824 = 14049\]

Thus, the tens digit is 4.
Stop!

Do not continue until directed to do so!
Tie-Breaker #2

(1 point)
If a man died 1 billion seconds after he was born, what age in years was printed on his death certificate?

Solution:

$$1 \times 10^9 \text{ sec} \times \frac{1 \text{ min}}{60 \text{ sec}} \times \frac{1 \text{ hr}}{60 \text{ min}} \times \frac{1 \text{ day}}{24 \text{ hrs}} \times \frac{1 \text{ years}}{365.25 \text{ days}} = 31.7 \text{ years}$$

Thus, he is 31 years old when he dies.
If a man died 1 billion seconds after he was born, what age in years was printed on his death certificate?

**Answer:**

To find the age in years, we need to convert 1 billion seconds into years. We can do this by dividing by the number of seconds in a year. The formula is:

\[
\text{Age in years} = \frac{1 \text{ billion seconds}}{1 \text{ year}} = \frac{1 \times 10^9 \text{ seconds}}{365 \times 24 \times 60 \times 60 \text{ seconds/years}}
\]

Calculating this gives:

\[
\text{Age in years} = \frac{1 \times 10^9}{365 \times 24 \times 60 \times 60} = 31.74 \text{ years}
\]

Therefore, he is approximately 31 years old when he dies.
Tie-Breaker #2
(1 point)

If a man died 1 billion seconds after he was born, what age in years was printed on his death certificate?

**Answer:** 31
If a man died 1 billion seconds after he was born, what age in years was printed on his death certificate?

Answer: 31

Solution:

\[
1 \times 10^9 \text{ sec} \times \frac{1 \text{ min}}{60 \text{ sec}} \times \frac{1 \text{ hr}}{60 \text{ min}} \times \frac{1 \text{ day}}{24 \text{ hrs}} \times \frac{1 \text{ years}}{365.25 \text{ days}} = 31.7 \text{ years}
\]

Thus, he is 31 years old when he dies.
Stop!

Do not continue until directed to do so!
Tie-Breaker #3

(1 point)
The smallest of three numbers is log 1. Their mean is log 4. Their median is log 5. Find the largest number.

Solution:

Let $x < y < z$. It follows that $y = \log 5$ (it is the median). Since their average is log 4, it follows that

$$\log 1 + \log 5 + z = \frac{\log 4}{3} \Rightarrow z = \log \left( \frac{64}{5} \right)$$
The smallest of three numbers is \( \log 1 \). Their mean is \( \log 4 \). Their median is \( \log 5 \). Find the largest number.

**Answer:**

\[
\log \left( \frac{64}{5} \right)
\]
The smallest of three numbers is $\log 1$. Their mean is $\log 4$. Their median is $\log 5$. Find the largest number.

**Answer:**

$$\log \left( \frac{64}{5} \right)$$
The smallest of three numbers is $\log 1$. Their mean is $\log 4$. Their median is $\log 5$. Find the largest number.

**Answer:**

$log \left( \frac{64}{5} \right)$

**Solution:**

Let $x < y < z$. It follows that $y = \log 5$ (it is the median). Since their average is $\log 4$, it follows that

$$\frac{\log 1 + \log 5 + z}{3} = \log 4$$

$$z + \log 5 = \log 4^3 \Rightarrow z = \log \left( \frac{64}{5} \right)$$