If $d < 0$ and the equation $9x^2 + dx + 4 = 0$ has only one solution, then what is that solution?
Two bicyclists, Kimo and Lani, are 30 miles apart on a steep road. Kimo and Lani travel at a constant speed and start riding towards each other at the same time. Kimo travels downhill and goes twice as fast as Lani. They expect to meet in one hour, but after only 30 minutes, Kimo’s bicycle breaks and he decides to sit and wait for Lani. How many more minutes should Kimo expect to wait for Lani if Lani continues at the same speed?
A very thin disk has an area (on one side) of $4\pi$. A square hole is cut into a surface. What is the smallest area the hole can have and still be large enough for the disk to fit through?

Answer:
On \((-\infty, \infty), h(x)\) is a straight line. On \([2, 4], h(x) = 2 + |x - 5| + 3|x + 2|\). At what point does the graph of \(h(x)\) cross the \(y\)-axis?

Answer:
The operation of addition is defined on the set of colors \{white, black, red, orange, yellow, green, blue, purple\} as follows:

\[
\begin{align*}
\text{black} + \text{color } x &= \text{color } x + \text{black} = \text{color } x \\
\text{red} + \text{blue} &= \text{blue} + \text{red} = \text{purple} \\
\text{red} + \text{yellow} &= \text{yellow} + \text{red} = \text{orange}
\end{align*}
\]

\[
\begin{align*}
\text{yellow} + \text{blue} &= \text{blue} + \text{yellow} = \text{green} \\
\text{color } x + \text{color } x &= \text{black} \\
\text{red} + \text{blue} + \text{yellow} &= \text{white}
\end{align*}
\]

Addition of colors is associative but not necessarily commutative. Under these rules what would “green + white” equal?

Answer: 

Graded By: ____________________________
Checked By: ____________________________
A certain cube has edges of length 1. A regular octahedron (a polyhedron with 8 faces) is inscribed in the cube such that the vertices of the octahedron are at the centers of the faces of the cube. Another regular octahedron is circumscribed about the cube such that the centers of the faces of the octahedron are at the vertices of the cube. What is the ratio of the length of a side of the larger octahedron to that of the smaller octahedron?
Define $\log^*(n)$ to be the smallest number of times the log(base 10) function must be iteratively applied to $n$ to get a result less than or equal to 1. For example $\log^*(1000) = 2$ since $\log(1000) = 3$ and $\log(3) = 0.477 \leq 1$. Let $a$ be the smallest integer such that $\log^*(a) = 3$. How many zeroes are in the base 10 representation of $a$?

Answer:

Graded By: ___________________________  Checked By: ___________________________

In $\triangle ABC$, $\tan(\angle CAB) = \frac{22}{7}$ and the altitude from $A$ divides $BC$ into segments of length 3 and 17. What is the area of $\triangle ABC$?
A factor of 243,000,000 is chosen at random. What is the probability that the factor is a multiple of 4?
Koch’s curve is created by starting with a line segment of length one. Call this stage 0. To get from one stage to the next we divide each line segment into thirds and replace the middle third by two line segments of the same length, according to the image. What is the length of Koch’s curve at stage 5?

Answer:

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State Math Bowl XXXVIII – 2016

Round #7, Problem A: (4 points/15 minutes)

Name of School (Please Print): ________________________________

(Please do not go below this line until directed to do so.)

All answers must be expressed exactly

Express $i^{1978} + i^{1979} + i^{1980} + i^{1981} + \cdots + i^{2016}$ as a single complex number, where $i^2 = -1$

Answer: ________________________________

Graded By: ________________________________  Checked By: ________________________________
An octagon (all sides not necessarily of the same length) in the plane is symmetric about the $x$-axis, the $y$-axis, and the line whose equation is $y = x$. If $(1, \sqrt{3})$ is a vertex of the octagon, find its area.
Consider the sequence with terms 2, 10, 6, 8, 7, $\frac{15}{2}$, $\frac{29}{4}$, ..., where each term in the sequence is the average of the preceding two. What is the largest real number smaller than infinitely many terms of the sequence?

Answer: 

Graded By: ___________________________  Checked By: ___________________________
Given that the area of the outer circle is ten square units, find the area of any one of the three equal circles inscribed inside it.
Cards numbered 1 to 100, inclusive, are shuffled, and four cards are drawn. What is the probability that these cards were drawn in increasing order?
Suppose you are on vacation and there are 4 cities, A, B, C, and D, you wish to visit. You want to spend the least time traveling between the cities as to maximize your time in each city. The travel times between the cities are given in the chart below.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0 min</td>
<td>15 min</td>
<td>25 min</td>
<td>30 min</td>
</tr>
<tr>
<td>B</td>
<td>15 min</td>
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<td>40 min</td>
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<tr>
<td>C</td>
<td>25 min</td>
<td>40 min</td>
<td>0 min</td>
<td>50 min</td>
</tr>
<tr>
<td>D</td>
<td>30 min</td>
<td>10 min</td>
<td>50 min</td>
<td>0 min</td>
</tr>
</tbody>
</table>

Assuming you can start and stop in any city or cities you wish, what is the minimum travel time you need to spend in between cities in order to make it to all four cities?

Answer:
Assume $f, g, h,$ and $u$ are invertible functions and that $f(4) = 3$, $g(4) = 7$, $h(7) = 3$, and $u(3) = 2$. Find $f(g^{-1}(7)) + u(h(7))$. 

Answer: 

Graded By: ___________________________  Checked By: ___________________________